Dimensional Analysis

Need For Dimensional Analysis

1. Reduce the number of variables under investigation.
2. Reduce the experimental work.
3. Consequently, reduce the effort and cost.

Dimension and Equations

For most engineering problems the basic dimensions are:

1. Length (L).
2. Mass (M).
3. Time (T).
4. Temperature (C).

Examples:

$$[F] = M\frac{L}{T^2}$$

$$[\mu] = \frac{FT}{L^2} = \frac{M}{LT}$$

$$[p] = \left[\frac{F}{A}\right] = \frac{M}{LT^2}$$

Buckingham pi Theorem

For a given Process:

Consider the number of variables = n

The number of basic dimensions for this process = m

Then the number of dimensionless groups = **(n - m)**

If the equation describing a physical system has (n) dimensional variables and is expressed:

$$y_1 = f(y_2, y_3, \ldots y_n)$$

then it can be rearranged and expressed in terms of $n-m$ dimensionless parameters (π-groups) as

$$\pi_1 = \varphi(\pi_2, \pi_3, \ldots \pi_{n-m})$$

Example:

Fluid passing a sphere.

The main variables are $F_D = f(F, V, \rho, \mu, D)$ n=5

The basic dimensions are (L, M, T) m=3

Number of PI groups = (n – m =3)

Methods of Dimensional Analysis

1. **The Step-by-Step Method**

In this method , we eliminate the basic dimension step by step.

Example (8.2):

If the drag F_D of a sphere in a fluid flowing past the sphere is a function of the viscosity μ, the mass density ρ, the velocity of flow V, and the diameter of the sphere D, what dimensionless parameters are applicable to the flow process?

$$F_D = f(V, \rho, \mu, D)$$

n=5

M=3

Pi groups = m – n = 2

Variable	[ ]	Variable	[ ]	Variable	[ ]	Variable	[]
F_D	$\frac{ML}{T^2}$	$\frac{F_D}{D}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho D^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 D^2}$	0
V	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρD^3	M				
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho VD}$	0
D	L						

$$\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho VD}\right)$$

2. The Exponent Method

The problem described in Example 8.2 is solved here using the exponent method. The dimensions of the variables are

$$[F] = \frac{ML}{T^2}$$

$$[V] = \frac{L}{T}$$

$$[\rho] = \frac{M}{L^3}$$

$$[\mu] = \frac{M}{LT}$$

$$[D] = L$$

$$[F] = [V^a \rho^b \mu^c D^d]$$

or

$$\frac{ML}{T^2} = \left(\frac{L}{T}\right)^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{LT}\right)^c L^d = \frac{L^{a-3b-c+d} M^{b+c}}{T^{a+c}}$$

Equating the powers of M, L, and T on each side of the equation results in three algebraic equations,

$$L:\quad a - 3b - c + d = 1$$

$$M:\quad b + c = 1$$

$$T:\quad a + c = 2$$

$$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ d \end{pmatrix} = \begin{pmatrix} 1 + c \\ 1 - c \\ 2 - c \end{pmatrix}$$

These exponents are now substituted back into the combination of the physical variables, and the result is

$$V^a \rho^b \mu^c D^d = \rho V^2 D^2 \left(\frac{\mu}{\rho V D}\right)^c$$

The factor $\rho V^2 D^2$ has the dimensions of force and will be common to every term in the function. The combination $(\mu / \rho V D)$ is dimensionless. Thus Eq. (8.7) can be rewritten as

$$F = \rho V^2 D^2 f\left(\frac{\mu}{\rho V D}\right)$$

Dividing through by $\rho V^2 D^2$ yields the same dimensionless equation as Eq. (8.8).

$$\frac{F}{\rho V^2 D^2} = f\left(\frac{\mu}{\rho V D}\right)$$

Note that in the process of dimensional analysis, we did not need to evaluate the exponent c. We sought only the form of the dimensionless parameter. The functional form of f must be obtained from experiment.

Common dimensional Numbers

1. Reynolds number: (Turbulence in a pipe flow)

$$\mathrm{Re} = \frac{\rho V L}{\mu}$$

2. Mach number: (Compressible flow)

$$\mathrm{M} = \frac{V}{c}$$

3. Weber number: (Liquid atomization)

$$\mathrm{We} = \frac{\rho L V^2}{\sigma}$$

4. Froude number: (Fluid motion with density stratification)

$$\mathrm{Fr} = \frac{V}{\sqrt{(\Delta\gamma/\gamma) g L}}$$

Similitude

Similitude is the theory of predicting prototype performance from model observation in the Lab.

Types of Similitude

1. Geometric Similitude

$$L_r = \frac{l_m}{l_p}$$

2. Dynamic Similitude

$$F_r = \frac{F_m}{F_p}$$

THE END